

FIGURE 10-28

Basic construction and operation of an acousto-optic tunable filter. Here, λ_1 and λ_3 are selected to be dropped at a node and the other wavelengths pass through. (Adapted with permission from Smith et al.,⁷⁷ © 1990, IEEE.)

PROBLEMS 1,2,3,6,7,12,18

- 10-1. An optical transmission system is constrained to have 500-GHz channel spacings. How many wavelength channels can be utilized in the 1536-to-1556-nm spectral band?
- 10-2. A product sheet for a 2×2 single-mode biconical tapered coupler with a 40/60 splitting ratio states that the insertion losses are 2.7 dB for the 60-percent channel and 4.7 dB for the 40-percent channel.
 - (a) If the input power $P_0 = 200 \mu\text{W}$, find the output levels P_1 and P_2 .
 - (b) Find the excess loss of the coupler.
 - (c) From the calculated values of P_1 and P_2 , verify that the splitting ratio is 40/60.
- 10-3. Consider the coupling ratios as a function of pull lengths as shown in Fig. P10-3 for a fused biconical tapered coupler. The performances are given for 1310-nm and 1540-nm operation. Discuss the behavior of the coupler for each wavelength if its pull length is stopped at the following points: A, B, C, D, E, and each F.
- 10-4. Consider the 2×2 coupler shown in Fig. 10-6, where **A** and **B** are the matrices representing the field strengths of the input and output propagating waves, respectively. For a given input a_1 , we impose the condition that there is no power emerging from the second input port; that is, $a_2 = 0$. Find expressions for the transmissivity T and the reflectivity R in terms of the elements s_{ij} in the scattering matrix **S** given in Eq. (10-8).
- 10-5. A 2×2 waveguide coupler has $\kappa = 0.4 \text{ mm}^{-1}$, $\alpha = 0.06 \text{ mm}^{-1}$, and $\Delta\beta = 0$. How long should it be to make a 3-dB power divider? If that length is doubled, what fraction of the input power emerges from the second channel?
- 10-6. Suppose we have two 2×2 waveguide couplers (couplers A and B) that have identical channel geometries and spacings, and are formed on the same substrate material. If the index of refraction of coupler A is larger than that of coupler B, which device has a larger coupling coefficient κ ? What does this imply about the device lengths needed in each case to form a 3-dB coupler?

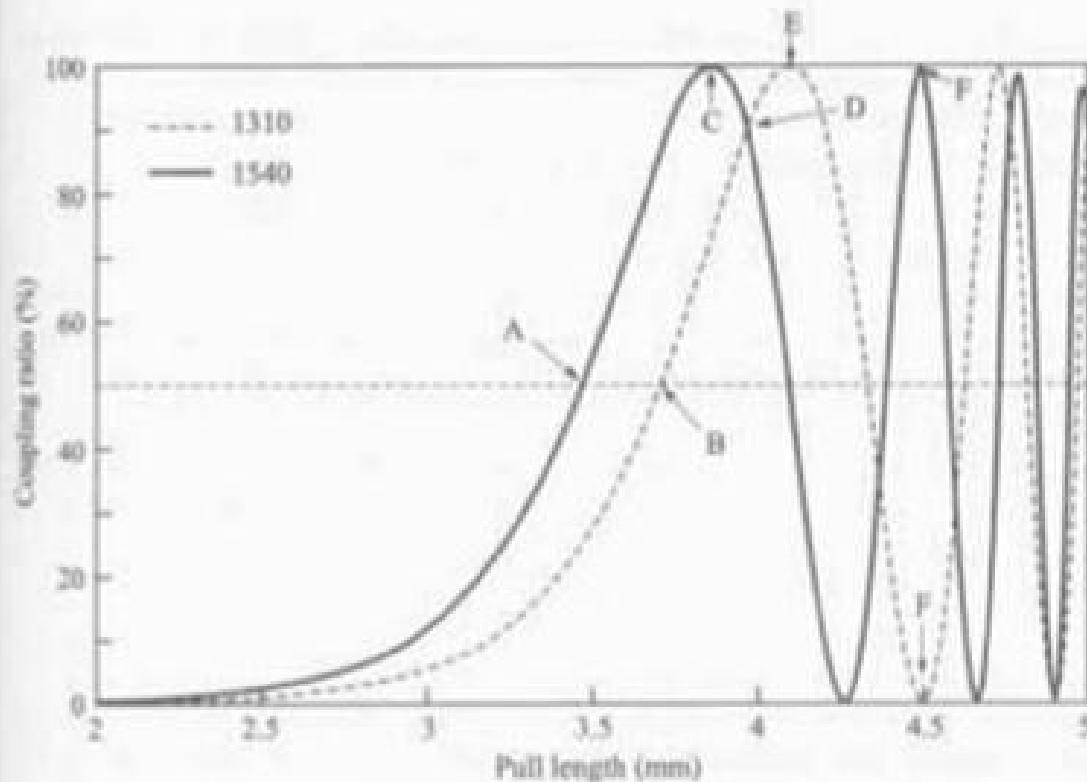


FIGURE P10-3

- 10-7. Measurements on a 7×7 star coupler yield the insertion losses from input port 1 to each output port shown in Table P10-7. Find the total excess loss through the coupler for inputs to port 1.

TABLE P10-7

Exit port no.	1	2	3	4	5	6	7
Insertion loss (dB)	9.33	7.93	7.53	9.03	9.63	8.64	9.04

- 10-8. Consider an optical fiber transmission star coupler that has seven inputs and seven outputs. Suppose the coupler is constructed by arranging the seven fibers in a circular pattern (a ring of six with one in the center) and butting them against the end of a glass rod that serves as the mixing element.
- If the fibers have $50\text{-}\mu\text{m}$ core diameters and $125\text{-}\mu\text{m}$ outer cladding diameters, what is the coupling loss resulting from light escaping between the fiber cores? Let the rod diameter be $300\text{-}\mu\text{m}$. Assume the fiber cladding is not removed.
 - What is the coupling loss if the fiber ends are arranged in a row and a $50\text{-}\mu\text{m} \times 800\text{-}\mu\text{m}$ glass plate is used as the star coupler?
- 10-9. Repeat Prob. 10-8 for seven fibers that have $200\text{-}\mu\text{m}$ core diameters and $400\text{-}\mu\text{m}$ outer cladding diameters. What should the sizes of the glass rod and the glass plate be in this case?
- 10-10. Suppose an $N \times N$ star coupler is constructed of n 3-dB 2×2 couplers, each of which has a 0.1-dB excess loss. Find the maximum value of n and the maximum size N if the power budget for the star coupler is 30 dB.

- 10-11. Using Eq. (10-29) for the 2×2 coupler propagation matrix, derive the expressions for M_{11} , M_{12} , M_{21} , and M_{22} in Eq. (10-35). From this, find the more general expressions for the output powers given by Eqs. (10-38) and (10-39).
- 10-12. Consider the 4×4 multiplexer shown in Fig. 10-14.
- (a) If $\lambda_1 = 1548$ nm and $\Delta\nu = 125$ GHz, what are the four input wavelengths?
- (b) If $n_{\text{eff}} = 1.5$, what are the values of ΔL_1 and ΔL_3 ?
- 10-13. Following the same line of analysis as in Example 10-6, use 2×2 Mach-Zehnder interferometers to design an 8-to-1 multiplexer that can handle a channel separation of 25 GHz. Let the shortest wavelength be 1550 nm. Specify the value of ΔL for the 2×2 MZIs in each stage.
- 10-14. A plane reflection grating can be used as a wavelength-division multiplexer when mounted as shown in Fig. P10-14. The angular properties of this grating are given by the grating equation

$$\sin \phi - \sin \theta = \frac{k\lambda}{n\Lambda}$$

where Λ is the grating period, k is the interference order, n is the refractive index of the medium between the lens and the grating, and ϕ and θ are the angles of the incident and reflected beams, respectively, measured normal to the grating.

(a) Using the grating equation, show that the angular dispersion is given by

$$\frac{d\theta}{d\lambda} = \frac{k}{n\Lambda \cos \theta} = \frac{2 \tan \theta}{\lambda}$$

(b) If the fractional beam spread S is given by

$$S = 2(1 + m) \frac{\Delta\lambda}{\lambda} \tan^2 \theta$$

where m is the number of wavelength channels, find the upper limit on θ for beam spreading of less than 1 percent given that $\Delta\lambda = 26$ nm, $\lambda = 1350$ nm, and $m = 3$.

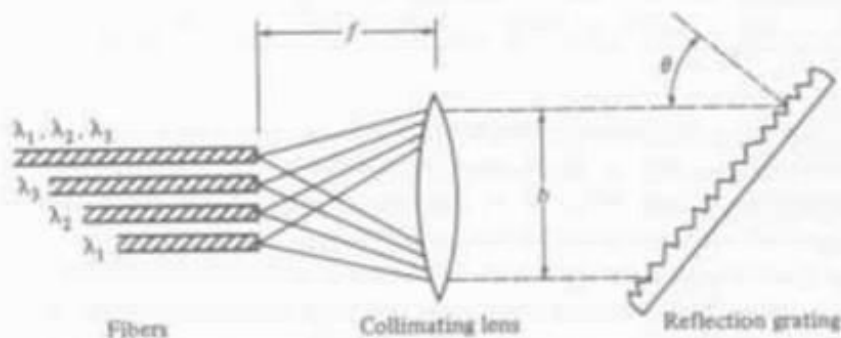


FIGURE P10-14

- 10-15. On the same graph, make plots of the reflectivity R_{max} given by Eq. (10-48) and the transmissivity $T = 1 - R_{\text{max}}$ for a fiber Bragg grating as a function of κL for $0 \leq \kappa L \leq 4$. If $\kappa = 0.75$ mm⁻¹, at what grating length does one get 93 percent reflectivity?

- 10-16. (This problem is best solved using a numerical program on a computer.) Based on coupled-mode theory, the reflectivity of a fiber grating is given by⁴²

$$R = \frac{(\kappa L)^2 \sinh^2(SL)}{(\delta\beta L)^2 \sinh^2(SL) + (SL)^2 \cosh^2(SL)} \quad \text{for} \quad (\kappa L)^2 > (\delta\beta L)^2$$

and

$$R = \frac{(\kappa L)^2 \sin^2(QL)}{(\delta\beta L)^2 - (\kappa L)^2 \cos^2(QL)} \quad \text{for} \quad (\kappa L)^2 < (\delta\beta L)^2$$

where

$$SL = (\delta\beta L) \left[\left(\frac{\kappa L}{\delta\beta L} \right)^2 - 1 \right]^{1/2} \quad \text{and} \quad QL = (\delta\beta L) \left[1 - \left(\frac{\kappa L}{\delta\beta L} \right)^2 \right]^{1/2}$$

Here, $\delta\beta = \beta - p\pi/\Lambda = 2\pi n_{\text{eff}}/\lambda - p\pi/\Lambda$, with Λ being the grating period and p an integer. For values of $\kappa L = 1, 2, 3$, and 4, plot $R(\kappa L)$ as a function of $\delta\beta L$ for the range $-10 \leq \delta\beta L \leq 10$. Note that R_{max} is found by setting $\delta\beta = 0$.

- 10-17. Using the expression for $R(\kappa L)$ given in Prob. 10-16, derive Eq. (10-49), which gives the full bandwidth $\Delta\lambda$ measured between the zeros on either side of R_{max} .
- 10-18. A 0.5-cm-long fiber Bragg grating is constructed by irradiating a single-mode fiber with a pair of 244-nm ultraviolet light beams. The fiber has $V = 2.405$ and $n_{\text{eff}} = 1.48$. The half-angle between the two beams is $\theta/2 = 13.5^\circ$. If the photo-induced index change is 2.5×10^{-4} , find the following:
- the grating period,
 - the Bragg wavelength,
 - the coupling coefficient,
 - the full bandwidth $\Delta\lambda$ measured between the zeros on either side of R_{max} ,
 - the maximum reflectivity.
- 10-19. Show that Eq. (10-55) follows from the differentiation of Eq. (10-53) with respect to frequency.
- 10-20. Consider a waveguide grating multiplexer that has the values for the operational variables listed in Table P10-20.
- Find the waveguide length difference.
 - Calculate the channel spacing $\Delta\nu$ and the corresponding pass wavelength differential $\Delta\lambda$.

TABLE P10-20

Symbol	Parameter	Value
L_f	Focal length	9.38 mm
λ_0	Center wavelength	1554 nm
n_c	Array channel index	1.451
n_g	Group index for n_c	1.475
n_s	Slab waveguide index	1.453
x	Input/output waveguide spacing	25 μm
d	Grating waveguide spacing	25 μm
m	Diffraction order	118